

西南大学附属中学高 2025 届阶段性检测（一）数学参考答案

1	2	3	4	5	6	7	8	9	10	11	12
C	B	D	C	A	B	C	D	ACD	ABC	AB	ABD

13. 5 14. 0 或 1 15. $\{x | 2\sqrt{2} - 1 < x \leq 2 \text{ 或 } x \geq 2\sqrt{2} + 1\}$ 16. $\frac{5}{4} + \frac{2\sqrt{3}}{9}$

17. (1) $A \cap B = \{x | 1 \leq x \leq 2\}$

(2) $\complement_U A = \{x | x < 1 \text{ 或 } 2 < x \leq 5\}$, $(\complement_U A) \cup B = \{x | x \leq 5\}$.

(3) $(\complement_U A) \cap (\complement_U B) = \complement_U (A \cup B) = \{x | x < -1 \text{ 或 } 4 < x \leq 5\}$.

18. (1) $A = \{-4, 0\}$,

$a = 0$, 可得 $B = \{1 - \sqrt{2}, 1 + \sqrt{2}\}$,

$A \cup B = \{-4, 0, 1 - \sqrt{2}, 1 + \sqrt{2}\}$

(2) ① $a^2 - 1 = 0$ 即 $a = \pm 1$

1) $a = 1, B = \left\{-\frac{1}{4}\right\}$, 舍去.

2) $a = -1, B = \{x | x^2 + 1 = 0\} = \emptyset$, 满足题意.

② $a^2 - 1 \neq 0$,

由 $B \subsetneq A$, 得 $\Delta = 4(a+1)^2 - 4(a^2 - 1) = 8a + 8 = 0$, 得 $a = -1$, 舍去.

综上, a 的取值范围为 $a = -1$.

19. (1) $m = 3$ 时, $P = \{x | -1 \leq x \leq 5\}$, $M \cap P = \{x | -1 \leq x \leq 4\}$.

(2) 由于 $m > 0$, $P = \{x | [x - (2 - m)][x - (2 + m)] \leq 0\} = \{x | 2 - m \leq x \leq 2 + m\}$

若选①, 由题得, $M \subsetneq P$, 即:

$$\begin{cases} 2 - m \leq -2 \\ 2 + m \geq 4 \end{cases}, \text{ 且等号不能同时成立,}$$

解得 $m \geq 4$

若选②, 由题得, $P \subsetneq M$, 即:

$$\begin{cases} 2-m \geq -2 \\ 2+m \leq 4 \end{cases}, \text{ 且等号不能同时成立,}$$

解得 $0 < m \leq 2$.

若选③, 由题得,:

$$\begin{cases} 2-m > -2 \\ 2+m > 4 \end{cases},$$

解得 $2 < m < 4$.

20. 原不等式等价于 $\begin{cases} [(a-2)x-2a+1](2x-1) \leq 0 \\ 2x-1 \neq 0 \end{cases},$

① $a = 2$, 解集为 $\left\{x \mid x > \frac{1}{2}\right\};$

② $a > 2$ 时, $[(a-2)x-2a+1](2x-1) = 0$ 的两根分别为 $x_1 = \frac{2a-1}{a-2}$, $x_2 = \frac{1}{2}$,

且 $x_1 > x_2$, 故解集为 $\left\{x \mid \frac{1}{2} < x \leq \frac{2a-1}{a-2}\right\};$

③ $a < 2$ 时,

$0 < a < 2$, 此时 $\frac{2a-1}{a-2} < \frac{1}{2}$, 解集为 $\left\{x \mid x \leq \frac{2a-1}{a-2} \text{ 或 } x > \frac{1}{2}\right\};$

$a = 0$ 时, 解集为 $\left\{x \mid x \neq \frac{1}{2}\right\};$

$a < 0$ 时, 此时 $\frac{2a-1}{a-2} > \frac{1}{2}$, 解集为 $\left\{x \mid x < \frac{1}{2} \text{ 或 } x \geq \frac{2a-1}{a-2}\right\}.$

综上: $a < 0$ 时, 解集为 $\left\{x \mid x < \frac{1}{2} \text{ 或 } x \geq \frac{2a-1}{a-2}\right\};$ $a = 0$ 时, 解集为 $\left\{x \mid x \neq \frac{1}{2}\right\};$

$0 < a < 2$, 此时 $\frac{2a-1}{a-2} < \frac{1}{2}$, 解集为 $\left\{x \mid x \leq \frac{2a-1}{a-2} \text{ 或 } x > \frac{1}{2}\right\};$ $a = 2$, 解集为 $\left\{x \mid x > \frac{1}{2}\right\};$

$a > 2$, 解集为 $\left\{x \mid \frac{1}{2} < x \leq \frac{2a-1}{a-2}\right\}.$

21. (1) 由题: $\begin{cases} a[1+(4x)\%](400-x) \geq 400a \\ 100 \leq x \leq 275 \end{cases},$ 解得: $\{x \in N \mid 100 \leq x \leq 275\}$

所以研发人员的人数最少为 125 人.

$$(2) \text{ 由题: } \begin{cases} a[1+(4x)\%](400-x) \geq a(m-\frac{2x}{25})x \\ a(m-\frac{2x}{25}) \geq a \end{cases} \text{ 恒成立,}$$

$$\text{即 } \begin{cases} a \geq \left(\frac{2x}{25}+1\right) \\ a \leq \left(\frac{x}{25}+\frac{400}{x}+15\right) \end{cases}$$

$$\text{故 } a \geq \left(\frac{2x}{25}+1\right)_{\max} = 23,$$

$$\text{且 } a \leq \left(\frac{x}{25}+\frac{400}{x}+15\right)_{\min},$$

$$\text{由基本不等式, } \frac{x}{25}+\frac{400}{x}+15 \geq 2\sqrt{\frac{x}{25} \cdot \frac{400}{x}}+15=23,$$

$$\text{当且仅当 } \frac{x}{25}=\frac{400}{x} \text{ 即 } x=100 \text{ 时取等,}$$

$$\text{即 } a \leq 23$$

$$\text{故 } a \text{ 的取值范围为 } a=23.$$

$$22. (1) \text{ 由题, } a+b+c=3$$

$$\begin{aligned} \text{故 } & \frac{1}{a+b} + \frac{1}{b+c} + \frac{1}{c+a} \\ &= \frac{1}{6} \times \left(\frac{1}{a+b} + \frac{1}{b+c} + \frac{1}{c+a} \right) [(a+b) + (b+c) + (a+c)] \\ &= \frac{1}{6} \left[3 + \left(\frac{b+c}{a+b} + \frac{a+b}{b+c} \right) + \left(\frac{a+c}{a+b} + \frac{a+b}{a+c} \right) + \left(\frac{a+c}{b+c} + \frac{b+c}{a+c} \right) \right] \\ &\geq \frac{1}{6} (3 + 2 \times 3) = \frac{3}{2}, \text{ 得证.} \end{aligned}$$

$$(2) \text{ 令 } a=1, b=-1, c=-1,$$

$$\text{方程 } x^2-x-1=0 \text{ 的两根为 } x_1=\frac{1-\sqrt{5}}{2}, x_2=\frac{1+\sqrt{5}}{2}$$

$$\text{由韦达定理, } x_1+x_2=1, x_1x_2=-1,$$

$$x_1^2+x_2^2=x_1(1-x_2)+x_2(1-x_1)=(x_1+x_2)-2x_1x_2=3$$

$$x_1^3+x_2^3=x_1^2(1-x_2)+x_2^2(1-x_1)=(x_1^2+x_2^2)-x_1x_2(x_1+x_2)=4$$

$$x_1^4 + x_2^4 = x_1^3(1-x_2) + x_2^3(1-x_1) = (x_1^3 + x_2^3) - x_1x_2(x_1^2 + x_2^2) = 7$$

$$x_1^5 + x_2^5 = x_1^4(1-x_2) + x_2^4(1-x_1) = (x_1^4 + x_2^4) - x_1x_2(x_1^3 + x_2^3) = 11$$

$$\text{同理, } x_1^6 + x_2^6 = 18, \quad x_1^7 + x_2^7 = 29, \quad x_1^8 + x_2^8 = 47, \quad x_1^9 + x_2^9 = 76, \quad x_1^{10} + x_2^{10} = 123,$$

又因为 $0 < x_1^{10} < 1$, 所以 $\left(\frac{1+\sqrt{5}}{2}\right)^{10}$ 的整数部分为 122.