

重庆市育才中学校高 2025 届 2022-2023 学年（下）3 月月考

参考答案

一、单选题：

1-4 : C D C C

5-8 : A D B B

二、多选题：

9.AD

10.ABD

11.ACD

12.AD

三、填空题：

13. (3,4)

14. $-\frac{3}{5}$

15. $\frac{\cos \frac{\pi}{2} x}{2}$

16. $[-\frac{5}{4}, -1], (-\infty, -\frac{1}{4}] \cup [\frac{5}{16}, +\infty)$

四、解答题：

17. (1) $(\frac{1}{2}, 0)$; (2) $\frac{\pi}{2}$.

解：(1) 由题意， $|\vec{a}|=2$ ， $|\vec{b}|=1$ ，设 $\frac{\vec{a}}{|\vec{a}|} = \vec{e}$ ， \vec{b} 在 \vec{a} 上的投影向量为 $|\vec{b}| \cdot \vec{e} \cdot \cos \langle \vec{a}, \vec{b} \rangle = 1 \times \frac{1}{2} \vec{e} = \frac{1}{2} \vec{e}$ ，

所以 \vec{b} 在 \vec{a} 上的投影向量的坐标为 $(\frac{1}{2}, 0)$4分

$$\begin{aligned} (2) |\vec{c}| &= \sqrt{(\vec{a} - t\vec{b})^2} = \sqrt{\vec{a}^2 - 2t\vec{a}\vec{b} + t^2\vec{b}^2} = \sqrt{|\vec{a}|^2 - 2t|\vec{a}||\vec{b}|\cos \langle \vec{a}, \vec{b} \rangle + t^2|\vec{b}|^2} \\ &= \sqrt{4 - 2t + t^2} = \sqrt{(t-1)^2 + 3} \geq \sqrt{3}, (t=1 \text{ 时等号成立}), \vec{c} = \vec{a} - \vec{b}, \quad (t \in \mathbb{R}), \end{aligned}$$

.....8分

所以 $\cos \langle \vec{b}, \vec{c} \rangle = \frac{\vec{b} \cdot (\vec{a} - \vec{b})}{|\vec{b}| \cdot |\vec{a} - \vec{b}|} = 0$ ，当 $|\vec{c}|$ 最小时， \vec{b} 与 \vec{c} 的夹角的大小为 $\frac{\pi}{2}$10分

法二：也可由坐标法写出 $\vec{a} = (2, 0)$ ， $\vec{b} = (\frac{1}{2}, \pm \frac{\sqrt{3}}{2})$ ， $\vec{c} = (\frac{3}{2}, \mp \frac{\sqrt{3}}{2})$ ，

$$\cos \langle \vec{b}, \vec{c} \rangle = \frac{\frac{1}{2} \times \frac{3}{2} + (\pm \frac{\sqrt{3}}{2}) \times (\mp \frac{\sqrt{3}}{2})}{1 \times \sqrt{3}} = 0,$$

得所求夹角为 $\frac{\pi}{2}$. 最值由图像分析得出最小值的结论也可酌情给分.

18. (1) $(\frac{\sqrt{3}}{2}, \sqrt{3}]$; (2) $A (\frac{2\sqrt{2}-\sqrt{3}}{6}, \frac{2\sqrt{6}+1}{6})$.

解：(1) 由题意 $A (\cos \alpha, \sin \alpha)$ ， $B (\cos(\alpha + \frac{\pi}{6}), \sin(\alpha + \frac{\pi}{6}))$ ， $x_1 + y_2 = \cos \alpha + \sin(\alpha + \frac{\pi}{6})$

$$= \cos \alpha + \frac{\sqrt{3}}{2} \sin \alpha + \frac{1}{2} \cos \alpha = \sqrt{3} (\frac{1}{2} \sin \alpha + \frac{\sqrt{3}}{2} \cos \alpha) = \sqrt{3} \sin(\alpha + \frac{\pi}{3}),$$

.....4分

由 $\alpha \in (0, \frac{\pi}{2})$ 可得 $\alpha + \frac{\pi}{3} \in (\frac{\pi}{3}, \frac{5\pi}{6})$, $\sin(\alpha + \frac{\pi}{3}) \in (\frac{1}{2}, 1]$, $x_1 + y_2$ 的取值范围是 $(\frac{\sqrt{3}}{2}, \sqrt{3}]$6分

(2) 由 $B(-\frac{1}{3}, \frac{2\sqrt{2}}{3})$, 得 $\cos(\alpha + \frac{\pi}{6}) = -\frac{1}{3}, \sin(\alpha + \frac{\pi}{6}) = \frac{2\sqrt{2}}{3}$,

$$\cos \alpha = \cos[(\alpha + \frac{\pi}{6}) - \frac{\pi}{6}] = \cos(\alpha + \frac{\pi}{6}) \cos \frac{\pi}{6} + \sin(\alpha + \frac{\pi}{6}) \sin \frac{\pi}{6} = (-\frac{1}{3}) \times \frac{\sqrt{3}}{2} + \frac{2\sqrt{2}}{3} \times \frac{1}{2} = \frac{2\sqrt{2} - \sqrt{3}}{6},$$

$$\sin \alpha = \sin[(\alpha + \frac{\pi}{6}) - \frac{\pi}{6}] = \sin(\alpha + \frac{\pi}{6}) \cos \frac{\pi}{6} - \cos(\alpha + \frac{\pi}{6}) \sin \frac{\pi}{6} = \frac{2\sqrt{2}}{3} \times \frac{\sqrt{3}}{2} - (-\frac{1}{3}) \times \frac{1}{2} = \frac{2\sqrt{6} + 1}{6},$$

所以点 A 的坐标为 $(\frac{2\sqrt{2} - \sqrt{3}}{6}, \frac{2\sqrt{6} + 1}{6})$12分

19. (1) 证明: 必要性, $\because P, M, N$ 三点共线, 不妨设 $\overrightarrow{MP} = y\overrightarrow{MN}$, 可得 $\overrightarrow{OP} - \overrightarrow{OM} = y(\overrightarrow{ON} - \overrightarrow{OM})$,

$$\overrightarrow{OP} = (1-y)\overrightarrow{OM} + y\overrightarrow{ON}, \text{ 又 } \because \overrightarrow{OP} = x\overrightarrow{OM} + y\overrightarrow{ON}, \therefore x = 1-y, \text{ 得 } x+y=1, \text{ 得证. } \dots 3\text{分}$$

充分性: $\because \overrightarrow{OP} = x\overrightarrow{OM} + y\overrightarrow{ON}, x+y=1, \therefore \overrightarrow{OP} = (1-y)\overrightarrow{OM} + y\overrightarrow{ON}, \therefore \overrightarrow{OP} - \overrightarrow{OM} = y(\overrightarrow{ON} - \overrightarrow{OM})$,

$$\therefore \overrightarrow{MP} = y\overrightarrow{MN}, \therefore P, M, N \text{ 三点共线. } \dots 6\text{分}$$

(2) 法一 (向量法)

证明: $\because \triangle ABC$ 的重心 G 是三条中线 AD, BE, CF 的交点, 由平面向量基本定理 $\overrightarrow{AG} = (1-y)\overrightarrow{AB} + y\overrightarrow{AE}$,

$$\overrightarrow{AD} = \frac{1}{2}\overrightarrow{AB} + \frac{1}{2}\overrightarrow{AC} = \frac{1}{2}\overrightarrow{AB} + \overrightarrow{AE}, \because A, G, D \text{ 三点共线}, \therefore \frac{1-y}{\frac{1}{2}} = \frac{y}{1}, y = \frac{2}{3}, \therefore \overrightarrow{AG} = \frac{2}{3}\overrightarrow{AD},$$

$\therefore G$ 为 AD 的三等分点, 同理可证 G 为 BE, CF 的三等分点, \therefore 重心为中线的三等分点.12分

法二 (几何法): 可连接一条中位线, 由三角形中位线定理和三角形相似得证.

$$\text{连接 } EF, \because E, F \text{ 为所在边的中点}, \therefore EF \parallel \frac{1}{2}BC, \triangle EFG \text{ 与 } \triangle BCG \text{ 相似}, \therefore \frac{EF}{BC} = \frac{FG}{GC} = \frac{EG}{GB} = \frac{1}{2},$$

$$\text{易证 } \frac{FG}{FC} = \frac{EG}{EB} = \frac{DG}{DA} = \frac{1}{3}, \therefore \text{重心为中线的三等分点.}$$

20. (1) $[2k\pi - \frac{2\pi}{3}, 2k\pi + \frac{\pi}{3}], k \in Z$, 对称轴为 $x = \frac{\pi}{3} + k\pi, k \in Z$, (2) $[\sqrt{3}, 2)$.

$$\text{解: (1) } f(x) = (2\sqrt{3}\sin \frac{x}{2} + \cos \frac{x}{2})\cos \frac{x}{2} + \sin \frac{x}{2}(-\sin \frac{x}{2}) = \sqrt{3}\sin x + \cos x = 2\sin(x + \frac{\pi}{6}), \dots 2\text{分}$$

$$\text{令 } 2k\pi - \frac{\pi}{2} \leq x + \frac{\pi}{6} \leq 2k\pi + \frac{\pi}{2}, k \in Z, 2k\pi - \frac{2\pi}{3} \leq x \leq 2k\pi + \frac{\pi}{3}, k \in Z \text{ 此时函数 } f(x) \text{ 单调递增,}$$

$$\therefore \text{函数 } f(x) \text{ 单调递增区间为 } [2k\pi - \frac{2\pi}{3}, 2k\pi + \frac{\pi}{3}], k \in Z. \dots 4\text{分}$$

令, $x + \frac{\pi}{6} = \frac{\pi}{2} + k\pi$, 得 $x = \frac{\pi}{3} + k\pi, (k \in Z)$, 所以函数 $f(x)$ 的对称轴为 $x = \frac{\pi}{3} + k\pi, (k \in Z)$;6分

(2) ① $\because x \in [0, \frac{\pi}{2}]$, $\therefore x + \frac{\pi}{6} \in [\frac{\pi}{6}, \frac{2\pi}{3}]$, 由图像分析得 $f(x) = m$, 有两个不同的解,

则 $\frac{\sqrt{3}}{2} \leq \sin(x + \frac{\pi}{6}) < 1, \sqrt{3} \leq 2\sin(x + \frac{\pi}{6}) < 2, \therefore m \in [\sqrt{3}, 2)$9分

②因为 α, β 是方程 $2\sin(x + \frac{\pi}{6}) = m$ 的两个根, 所以 $2\sin(\alpha + \frac{\pi}{6}) = m, 2\sin(\beta + \frac{\pi}{6}) = m$,

由图像分析得, $\alpha + \beta = \frac{2\pi}{3}, \beta = \frac{2\pi}{3} - \alpha, \alpha - \beta = 2\alpha - \frac{2\pi}{3},$

$\cos(\alpha - \beta) = \cos(2\alpha - \frac{2\pi}{3}) = -\cos(2\alpha + \frac{\pi}{3}) = 2\sin^2(\alpha + \frac{\pi}{6}) - 1 = 2 \times (\frac{m}{2})^2 - 1 = \frac{m^2}{2} - 1.$ 12分

21.(1) $\{x | 0 < x < 1 \text{ 或 } 2 < x < 3\}; (2) [4, +\infty).$

解: (1) $f(3) = \log_2(3^2 - 3 \times 3 + a) = 1$, 解得 $a = 2$,2分

$f(x) = \log_2(x^2 - 3x + 2) < 1 = \log_2 2$, 得 $\begin{cases} x^2 - 3x + 2 > 0 \\ x^2 - 3x + 2 < 2 \end{cases}$, 解集为 $\{x | 0 < x < 1 \text{ 或 } 2 < x < 3\}$ 6分

(2) $\because a > 2, t \in [3, 4], \therefore x \in [t, t+1]$ 是函数 $f(x)$ 定义域的子集,

令 $p(x) = x^2 - 3x + a, p(x)$ 在 $[2, +\infty)$ 上单调递增,

由复合函数单调性知 $f(x)$ 在 $x \in [t, t+1]$ 上单调递增, $f(x)_{\max} = f(t+1), f(x)_{\min} = f(t)$,

由题意, $f(t+1) - f(t) \leq 1$, 即 $\log_2[(t+1)^2 - 3(t+1) + a] \leq \log_2 2(t^2 - 3t + a)$,

整理得: $a \geq -t^2 + 5t - 2$, 令 $g(t) = -t^2 + 5t - 2, g(t)_{\max} = g(3) = 4, \therefore a \geq 4$,

故 a 的取值范围是 $[4, +\infty)$12分

22.(1) $T_3(x) = 4x^3 - 3x; (2) \sin 18^\circ = \frac{\sqrt{5}-1}{4};$

解: (1) 因为 $\cos 3\theta = \cos(2\theta + \theta) = \cos 2\theta \cos \theta - \sin 2\theta \sin \theta$

$\cos 3\theta = (2\cos^2 \theta - 1)\cos \theta - 2\sin^2 \theta \cos \theta = 2\cos^3 \theta - \cos \theta - 2(1 - \cos^2 \theta)\cos \theta$

$= 4\cos^3 \theta - 3\cos \theta, \therefore T_3(x) = 4x^3 - 3x \dots 4分,$

(2) 因为, $\cos 54^\circ = \sin 36^\circ \Leftrightarrow 4\cos^3 18^\circ - 3\cos 18^\circ = 2\sin 18^\circ \cos 18^\circ,$

因为 $\cos 18^\circ > 0, 4\cos^2 18^\circ - 3 = 2\sin 18^\circ \Leftrightarrow 4(1 - \sin^2 18^\circ) - 3 = 2\sin 18^\circ,$

即 $4\sin^2 18^\circ + 2\sin 18^\circ - 1 = 0$, 因为 $\sin 18^\circ > 0$, 解得 $\sin 18^\circ = \frac{\sqrt{5}-1}{4}, (\frac{-\sqrt{5}-1}{4} \text{舍去})$;8分

(3) 由题意, $4x^3 - 3x - \frac{1}{2} = 0$,

法一: 设 $x = \cos \theta$, 代入方程得到 $4\cos^3 \theta - 3\cos \theta - \frac{1}{2} = 0 \Rightarrow \cos 3\theta = \frac{1}{2}$,

解三角方程得 $3\theta = \frac{\pi}{3} + 2k\pi, 3\theta = -\frac{\pi}{3} + 2k\pi, k \in Z$, 不妨取 $\theta_1 = \frac{\pi}{9}, \theta_2 = \frac{5\pi}{9}, \theta_3 = \frac{7\pi}{9}$,

$x_1 + x_2 + x_3 = \cos \frac{\pi}{9} + \cos \frac{5\pi}{9} + \cos \frac{7\pi}{9} = \cos \frac{\pi}{9} - (\cos \frac{4\pi}{9} + \cos \frac{2\pi}{9})$,

而 $\cos \frac{4\pi}{9} + \cos \frac{2\pi}{9} = \cos(\frac{3\pi}{9} + \frac{\pi}{9}) + \cos(\frac{3\pi}{9} - \frac{\pi}{9}) = \cos \frac{\pi}{3} = 1$, 综上 $x_1 + x_2 + x_3 = 0$12分

法二: 令, $4x^3 - 3x - \frac{1}{2} = 4(x - x_1)(x - x_2)(x - x_3) = 0$

即, $4[x^3 - (x_1 + x_2 + x_3)x^2 + (x_1x_2 + x_2x_3 + x_1x_3)x - x_1x_2x_3] = 4x^3 - 3x - \frac{1}{2} = 0$

依据多项式系数对应相等得到 $x_1 + x_2 + x_3 = 0$. 综上 $x_1 + x_2 + x_3 = 0$.